**Supplementary Appendix to**

# OUTPUT GAP ESTIMATION AND MONETARY POLICY WITH IMPERFECT KNOWLEDGE

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This supplementary appendix collects additional results that are not included in the paper or its online appendix. Section 1 reports the full version of Table A.8 of the paper, with optimized policy coefficients and welfare losses under a 4% inflation target and under a dual mandate for four degrees of interest rate inertia ($\theta_r = 0$, 0.2, 0.6 and 0.8). Sections 2–7 report, for each alternative scenario considered in the paper, the iso-loss contours for the degrees of interest rate inertia not shown in the online appendix. In each panel the horizontal axis is the policy response to inflation ($\theta_\pi$) and the vertical axis the response to the output gap estimate ($\theta_x$); the shaded region is stable (welfare loss below one). In every scenario and for every degree of inertia, the ranking of detrending methods by the size of the stability region is the same as in the paper.

## 1 Optimized policy coefficients for four degrees of inertia

Table S.1 extends Table A.8 of the paper (which reports $\theta_r = 0$ and 0.6) to all four degrees of interest rate inertia.

**Table S.1: OPTIMIZED POLICY COEFFICIENTS AND WELFARE LOSS: 4% INFLATION TARGET OR DUAL MANDATE**

| Panel A: 4% inflation target | | | | | | | Panel B: Dual mandate | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Method | Coeff | | SD | | | | | Coeff | | SD | | | |
| | $\theta_\pi$ | $\theta_x$ | $\sigma_\pi$ | $\sigma_x$ | *Loss* | ZLB(%) | | $\theta_\pi$ | $\theta_x$ | $\sigma_\pi$ | $\sigma_x$ | *Loss* | ZLB(%) |
| **Part 1: Methods intrinsically producing smaller and less volatile $\widetilde{x}_{t-1}$** | | | | | | | | | | | | | |
| Christiano-Fitzgerald band-pass filter | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.5 | 1.8 | 0.474 | 4.847 | 0.295 | 40.0 | $\theta_r = 0$ | 5.0 | 1.3 | 0.481 | 4.800 | 0.591 | 43.0 |
| $\theta_r = 0.2$ | 4.8 | 1.1 | 0.473 | 4.854 | 0.295 | 30.9 | $\theta_r = 0.2$ | 3.8 | 1.0 | 0.481 | 4.802 | 0.591 | 36.2 |
| $\theta_r = 0.6$ | 2.6 | 0.7 | 0.473 | 4.902 | 0.295 | 22.8 | $\theta_r = 0.6$ | 2.2 | 0.7 | 0.479 | 4.839 | 0.595 | 31.1 |
| $\theta_r = 0.8$ | 1.7 | 0.6 | 0.473 | 5.030 | 0.300 | 23.7 | $\theta_r = 0.8$ | 1.5 | 0.6 | 0.479 | 4.929 | 0.609 | 23.3 |
| Beveridge-Nelson (Kamber, Morley and Wong, 2018) | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.0 | 1.0 | 0.474 | 4.833 | 0.295 | 35.8 | $\theta_r = 0$ | 4.7 | 0.8 | 0.482 | 4.789 | 0.590 | 40.1 |
| $\theta_r = 0.2$ | 4.9 | 0.8 | 0.473 | 4.846 | 0.294 | 31.5 | $\theta_r = 0.2$ | 3.9 | 0.7 | 0.480 | 4.793 | 0.590 | 36.7 |
| $\theta_r = 0.6$ | 2.6 | 0.5 | 0.473 | 4.893 | 0.295 | 22.5 | $\theta_r = 0.6$ | 2.1 | 0.5 | 0.480 | 4.818 | 0.593 | 29.4 |
| $\theta_r = 0.8$ | 1.6 | 0.4 | 0.474 | 5.010 | 0.300 | 21.1 | $\theta_r = 0.8$ | 1.4 | 0.4 | 0.480 | 4.927 | 0.610 | 30.0 |
| HP filter | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.0 | 0.5 | 0.474 | 4.892 | 0.296 | 34.0 | $\theta_r = 0$ | 4.5 | 0.4 | 0.481 | 4.826 | 0.595 | 36.2 |
| $\theta_r = 0.2$ | 4.9 | 0.4 | 0.473 | 4.908 | 0.296 | 30.0 | $\theta_r = 0.2$ | 3.8 | 0.4 | 0.479 | 4.833 | 0.595 | 33.7 |
| $\theta_r = 0.6$ | 2.4 | 0.3 | 0.473 | 4.947 | 0.297 | 18.7 | $\theta_r = 0.6$ | 2.2 | 0.4 | 0.478 | 4.871 | 0.600 | 29.8 |
| $\theta_r = 0.8$ | 1.5 | 0.3 | 0.474 | 5.074 | 0.302 | 19.2 | $\theta_r = 0.8$ | 1.3 | 0.3 | 0.480 | 4.963 | 0.615 | 27.5 |
| First-order differencing | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.8 | 1.9 | 0.474 | 4.774 | 0.293 | 46.8 | $\theta_r = 0$ | 5.2 | 1.0 | 0.482 | 4.772 | 0.588 | 48.1 |
| $\theta_r = 0.2$ | 5.3 | 1.6 | 0.474 | 4.767 | 0.293 | 41.1 | $\theta_r = 0.2$ | 4.1 | 0.9 | 0.482 | 4.765 | 0.587 | 43.6 |
| $\theta_r = 0.6$ | 2.9 | 1.2 | 0.473 | 4.761 | 0.292 | 34.2 | $\theta_r = 0.6$ | 2.5 | 0.8 | 0.481 | 4.764 | 0.586 | 40.8 |
| $\theta_r = 0.8$ | 2.0 | 1.1 | 0.474 | 4.790 | 0.294 | 36.5 | $\theta_r = 0.8$ | 2.0 | 0.8 | 0.481 | 4.812 | 0.593 | 46.0 |
| Beveridge-Nelson (1981) | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.9 | 1.6 | 0.474 | 4.835 | 0.295 | 45.2 | $\theta_r = 0$ | 4.8 | 0.7 | 0.482 | 4.800 | 0.592 | 43.3 |
| $\theta_r = 0.2$ | 5.1 | 1.2 | 0.473 | 4.834 | 0.295 | 37.3 | $\theta_r = 0.2$ | 4.0 | 0.7 | 0.481 | 4.800 | 0.592 | 40.7 |
| $\theta_r = 0.6$ | 2.6 | 0.8 | 0.473 | 4.850 | 0.294 | 27.2 | $\theta_r = 0.6$ | 2.3 | 0.6 | 0.480 | 4.815 | 0.593 | 35.9 |
| $\theta_r = 0.8$ | 1.5 | 0.6 | 0.474 | 4.933 | 0.298 | 24.3 | $\theta_r = 0.8$ | 1.7 | 0.6 | 0.482 | 4.883 | 0.604 | 40.5 |
| **Part 2: Methods intrinsically producing larger and more volatile $\widetilde{x}_{t-1}$** | | | | | | | | | | | | | |
| Blanchard-Quah | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.0 | 0.1 | 0.473 | 4.909 | 0.296 | 33.0 | $\theta_r = 0$ | 4.7 | 0.1 | 0.479 | 4.841 | 0.596 | 36.8 |
| $\theta_r = 0.2$ | 4.7 | 0.1 | 0.473 | 4.899 | 0.296 | 27.2 | $\theta_r = 0.2$ | 3.6 | 0.1 | 0.479 | 4.846 | 0.596 | 30.0 |
| $\theta_r = 0.6$ | 2.4 | 0.0 | 0.473 | 4.977 | 0.298 | 16.7 | $\theta_r = 0.6$ | 1.9 | 0.0 | 0.478 | 4.897 | 0.603 | 20.3 |
| $\theta_r = 0.8$ | 1.3 | 0.0 | 0.474 | 5.112 | 0.303 | 12.0 | $\theta_r = 0.8$ | 1.1 | 0.0 | 0.478 | 5.018 | 0.622 | 17.1 |
| Beveridge-Nelson (bivariate) | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.0 | 0.1 | 0.473 | 4.909 | 0.296 | 33.0 | $\theta_r = 0$ | 4.7 | 0.1 | 0.479 | 4.841 | 0.596 | 36.8 |
| $\theta_r = 0.2$ | 4.7 | 0.1 | 0.473 | 4.899 | 0.296 | 27.2 | $\theta_r = 0.2$ | 3.6 | 0.1 | 0.479 | 4.846 | 0.596 | 30.0 |
| $\theta_r = 0.6$ | 2.4 | 0.0 | 0.473 | 4.977 | 0.298 | 16.7 | $\theta_r = 0.6$ | 1.9 | 0.0 | 0.478 | 4.897 | 0.603 | 20.3 |
| $\theta_r = 0.8$ | 1.3 | 0.0 | 0.474 | 5.112 | 0.303 | 12.0 | $\theta_r = 0.8$ | 1.1 | 0.0 | 0.478 | 5.018 | 0.622 | 17.1 |
| Hamilton | | | | | | | | | | | | | |
| $\theta_r = 0$ | 6.0 | 0.4 | 0.474 | 4.819 | 0.294 | 35.8 | $\theta_r = 0$ | 5.0 | 0.4 | 0.482 | 4.778 | 0.589 | 43.4 |
| $\theta_r = 0.2$ | 5.1 | 0.4 | 0.473 | 4.827 | 0.294 | 34.4 | $\theta_r = 0.2$ | 3.9 | 0.3 | 0.481 | 4.785 | 0.589 | 37.3 |
| $\theta_r = 0.6$ | 2.6 | 0.2 | 0.473 | 4.880 | 0.295 | 23.1 | $\theta_r = 0.6$ | 2.2 | 0.2 | 0.479 | 4.823 | 0.593 | 31.0 |
| $\theta_r = 0.8$ | 1.5 | 0.1 | 0.473 | 5.024 | 0.299 | 17.3 | $\theta_r = 0.8$ | 1.6 | 0.2 | 0.481 | 4.933 | 0.612 | 29.5 |
| Quadratic trend | | | | | | | | | | | | | |
| $\theta_r = 0$ | 5.8 | 0.2 | 0.474 | 4.889 | 0.296 | 32.7 | $\theta_r = 0$ | 4.5 | 0.1 | 0.480 | 4.828 | 0.595 | 35.6 |
| $\theta_r = 0.2$ | 4.6 | 0.1 | 0.473 | 4.900 | 0.296 | 26.9 | $\theta_r = 0.2$ | 3.6 | 0.1 | 0.479 | 4.834 | 0.595 | 30.8 |
| $\theta_r = 0.6$ | 2.5 | 0.1 | 0.473 | 4.963 | 0.297 | 19.4 | $\theta_r = 0.6$ | 2.1 | 0.1 | 0.478 | 4.892 | 0.602 | 25.8 |
| $\theta_r = 0.8$ | 1.3 | 0.0 | 0.474 | 5.112 | 0.303 | 12.0 | $\theta_r = 0.8$ | 1.1 | 0.0 | 0.478 | 5.018 | 0.622 | 17.1 |
| Linear trend | | | | | | | | | | | | | |
| $\theta_r = 0$ | 5.9 | 0.2 | 0.474 | 4.807 | 0.294 | 34.1 | $\theta_r = 0$ | 4.6 | 0.2 | 0.482 | 4.762 | 0.587 | 39.0 |
| $\theta_r = 0.2$ | 5.0 | 0.2 | 0.473 | 4.816 | 0.293 | 32.0 | $\theta_r = 0.2$ | 4.0 | 0.2 | 0.481 | 4.772 | 0.588 | 38.2 |
| $\theta_r = 0.6$ | 2.6 | 0.1 | 0.472 | 4.872 | 0.294 | 21.5 | $\theta_r = 0.6$ | 2.1 | 0.1 | 0.480 | 4.823 | 0.594 | 27.8 |
| $\theta_r = 0.8$ | 1.7 | 0.1 | 0.475 | 5.076 | 0.303 | 20.8 | $\theta_r = 0.8$ | 1.1 | 0.0 | 0.478 | 5.018 | 0.622 | 17.1 |
| Discounted Least Squares | | | | | | | | | | | | | |
| $\theta_r = 0$ | 7.0 | 0.1 | 0.474 | 4.872 | 0.296 | 40.6 | $\theta_r = 0$ | 5.4 | 0.1 | 0.483 | 4.805 | 0.594 | 46.0 |
| $\theta_r = 0.2$ | 6.1 | 0.1 | 0.473 | 4.888 | 0.296 | 38.8 | $\theta_r = 0.2$ | 4.7 | 0.1 | 0.483 | 4.810 | 0.595 | 44.6 |
| $\theta_r = 0.6$ | 2.4 | 0.0 | 0.473 | 4.977 | 0.298 | 16.7 | $\theta_r = 0.6$ | 1.9 | 0.0 | 0.478 | 4.897 | 0.603 | 20.3 |
| $\theta_r = 0.8$ | 1.3 | 0.0 | 0.474 | 5.112 | 0.303 | 12.0 | $\theta_r = 0.8$ | 1.1 | 0.0 | 0.478 | 5.018 | 0.622 | 17.1 |

*Notes:* This table reports optimized policy coefficients and welfare loss for each detrending method. Panel A presents results under a 4% inflation target; Panel B presents results under a dual mandate. SD denotes standard deviation from simulation.

# 2 Without ZLB

This section complements Appendix G.1 of the paper, which reports the iso-loss contours for the remaining degrees of inertia.

**Figure S.1: WITHOUT ZLB: ISO-LOSS CONTOURS, $\theta_r = 0.2$**

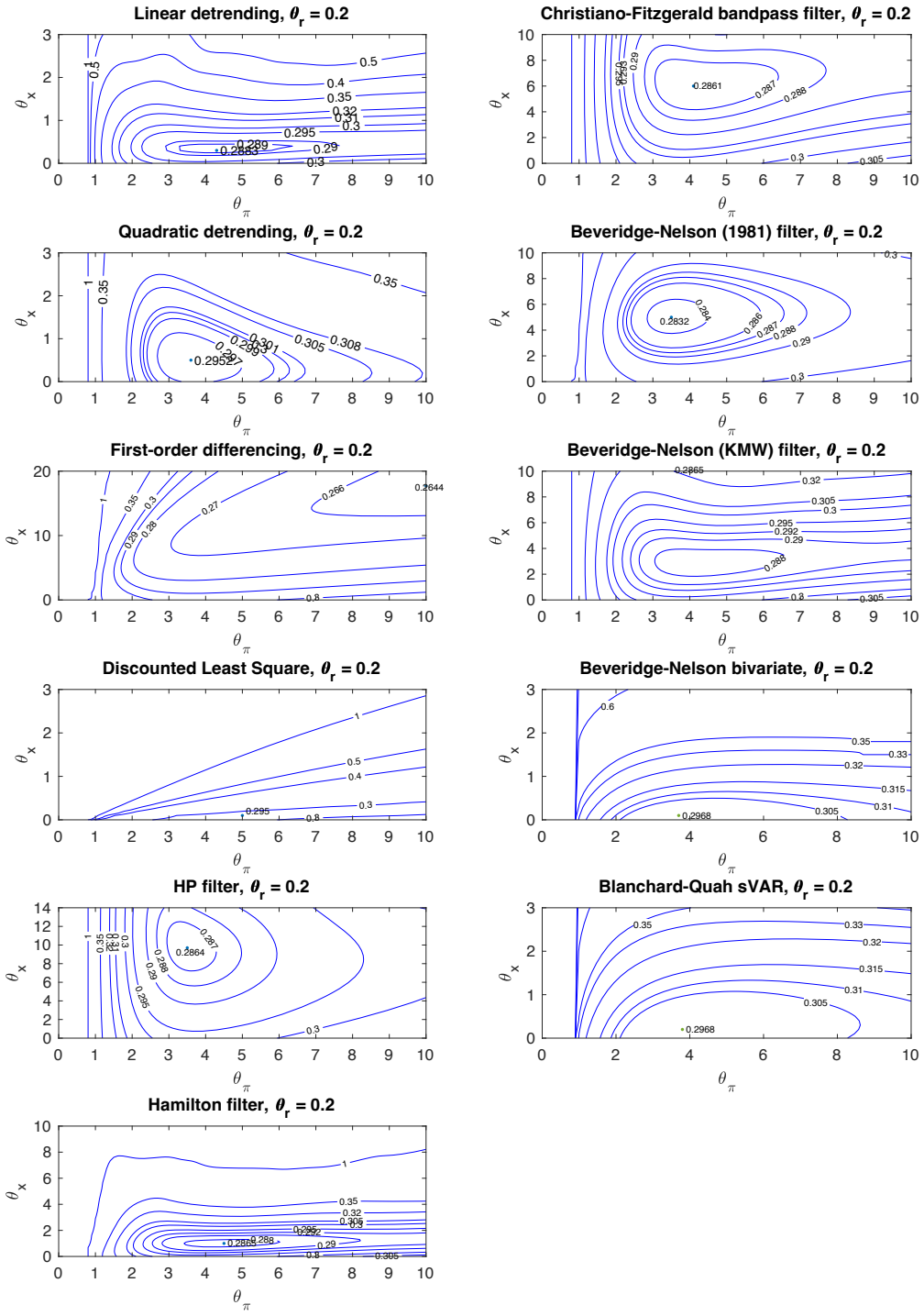
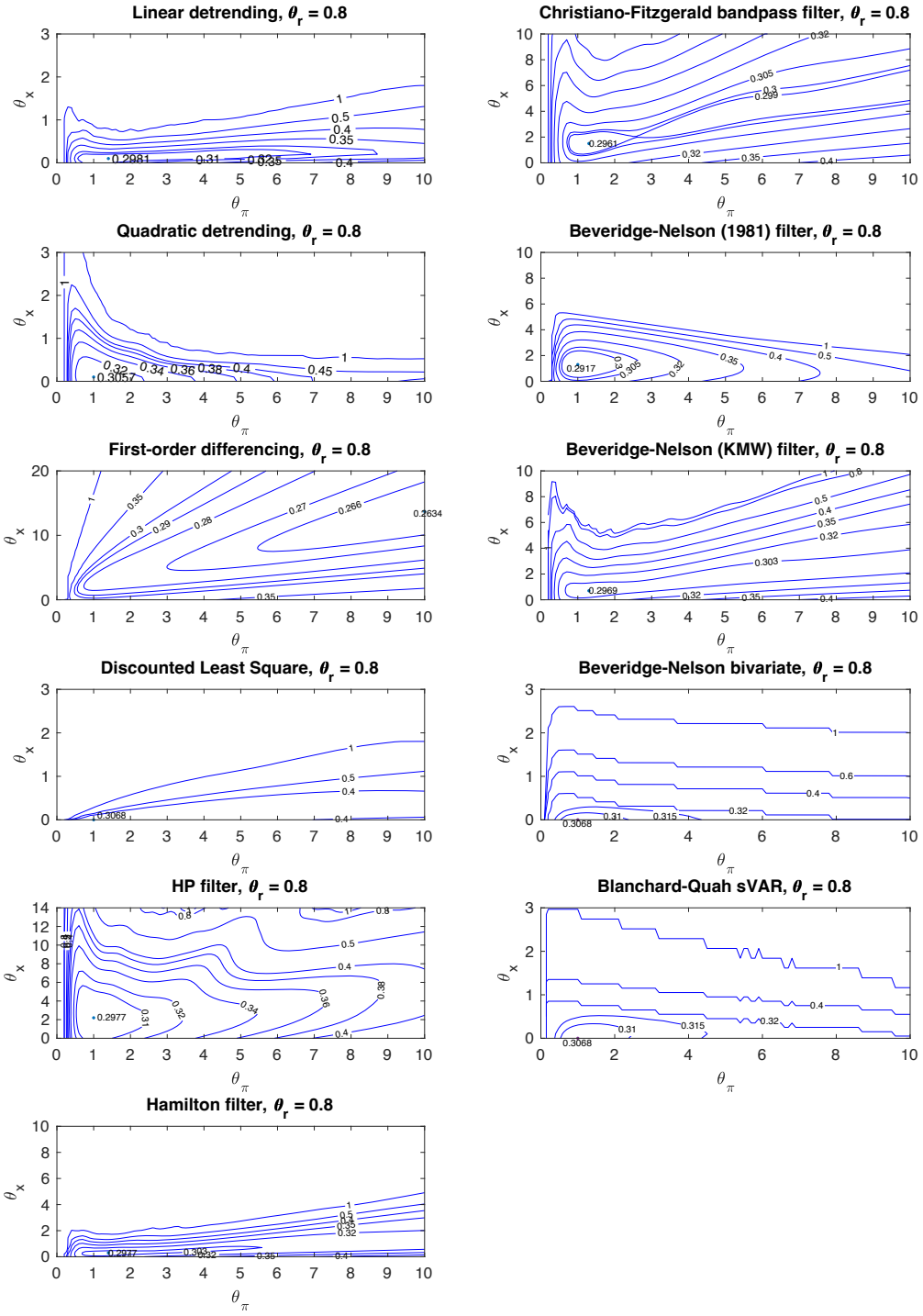


**Figure S.2: WITHOUT ZLB: ISO-LOSS CONTOURS, $\theta_r = 0.8$**




# 3 A −1% lower bound on nominal interest rates

This section complements Appendix G.2 of the paper, which reports the iso-loss contours for the remaining degrees of inertia.

**Figure S.3: $-1\%$ INTEREST RATE BOUND: ISO-LOSS CONTOURS, $\theta_r = 0.2$**

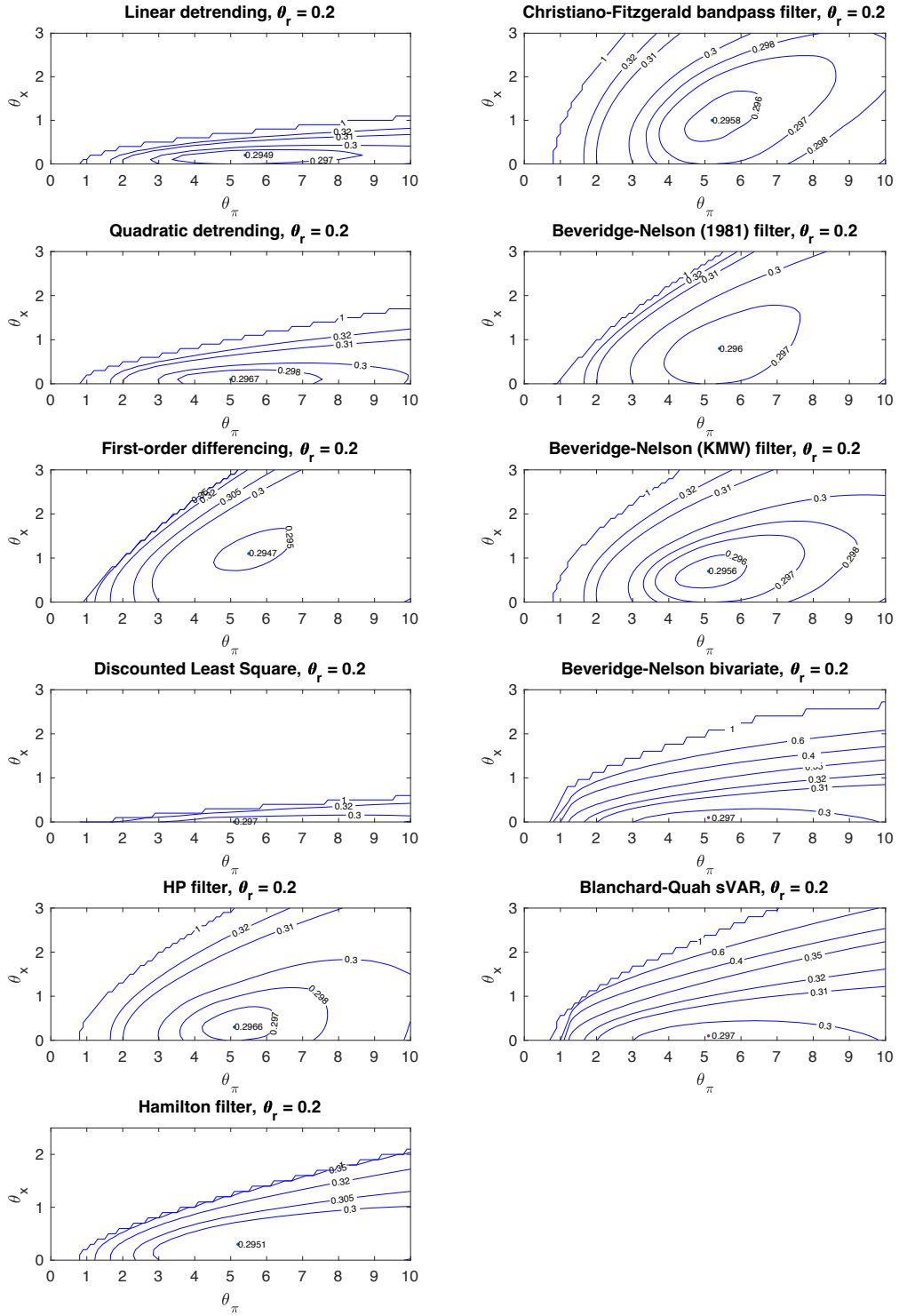


**Figure S.4: $-1\%$ INTEREST RATE BOUND: ISO-LOSS CONTOURS, $\theta_r = 0.6$**

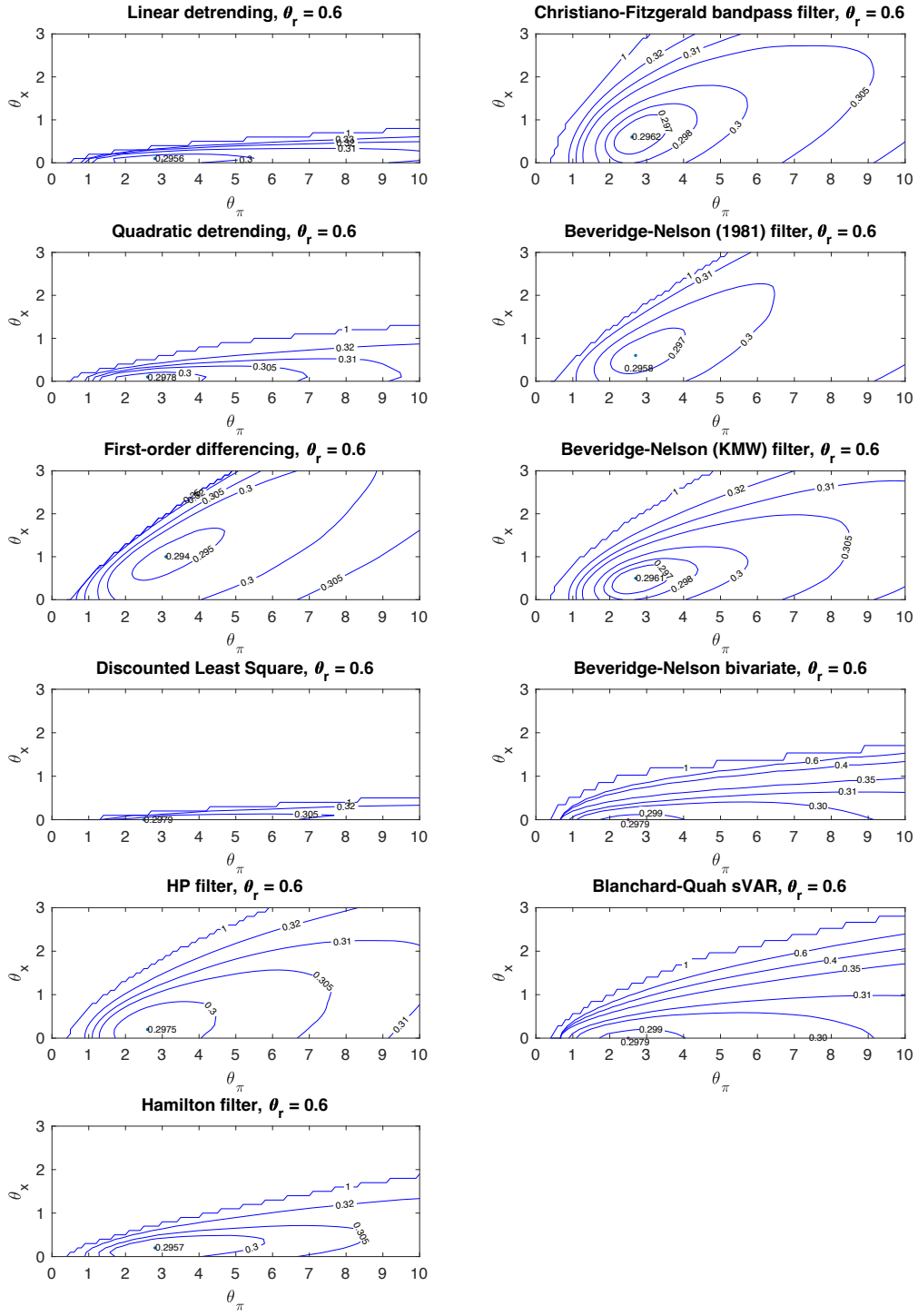
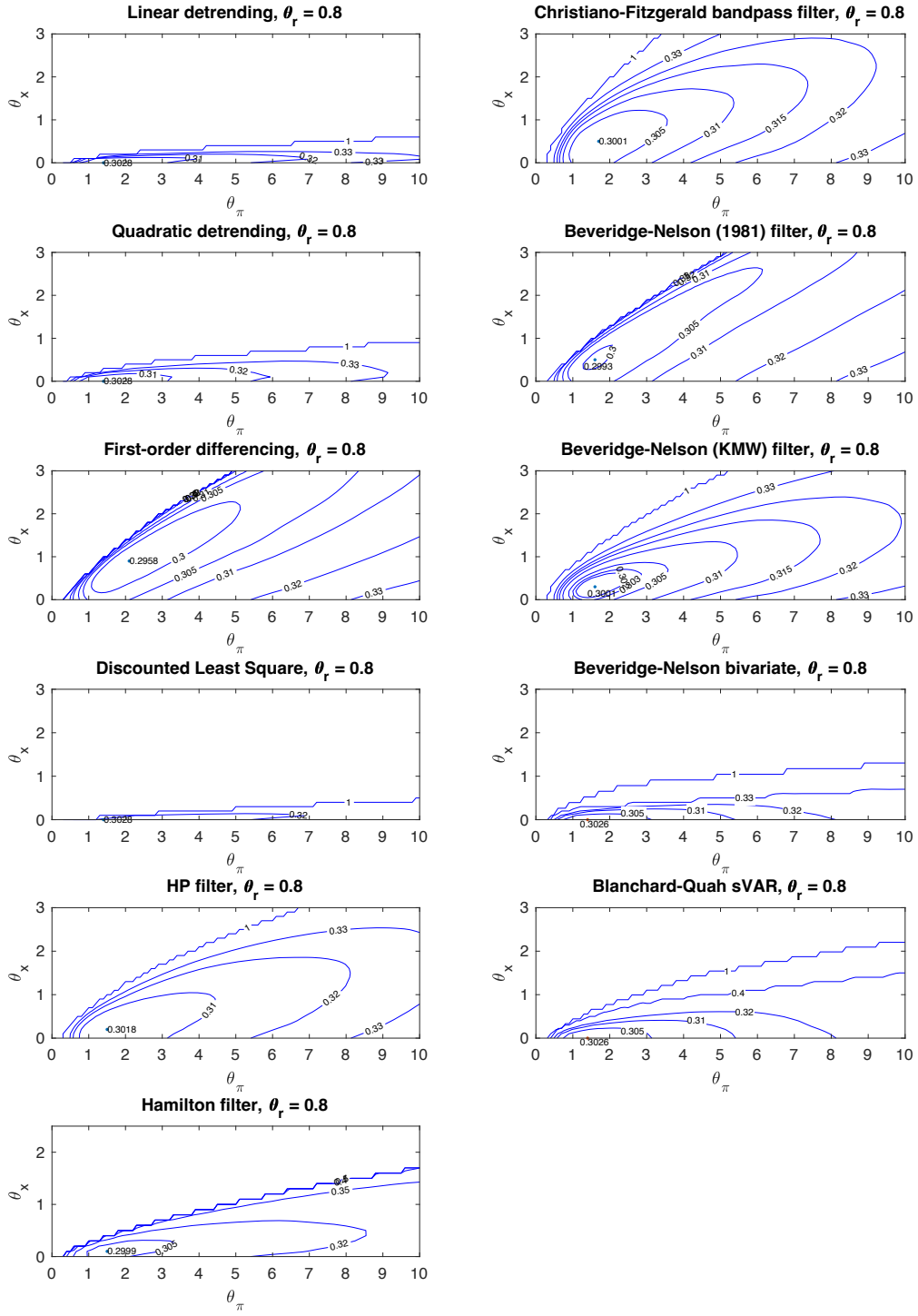


**Figure S.5: $-1\%$ INTEREST RATE BOUND: ISO-LOSS CONTOURS, $\theta_r = 0.8$**



# 4 A 4% inflation target

This section complements Appendix J of the paper, which reports the iso-loss contours for the remaining degrees of inertia.

**Figure S.6: 4% INFLATION TARGET: ISO-LOSS CONTOURS, $\theta_r = 0.2$**

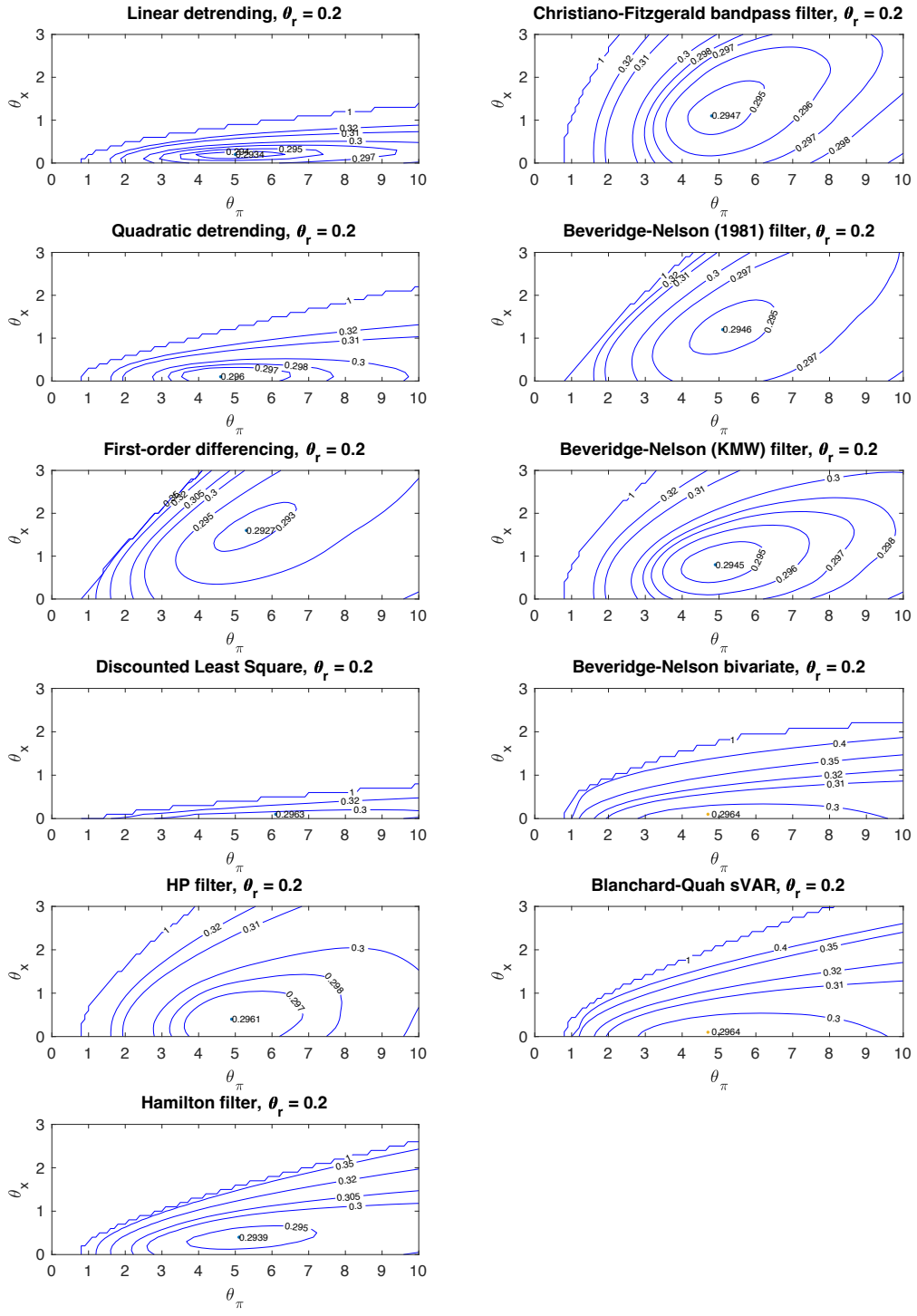


**Figure S.7: 4% INFLATION TARGET: ISO-LOSS CONTOURS, $\theta_r = 0.6$**

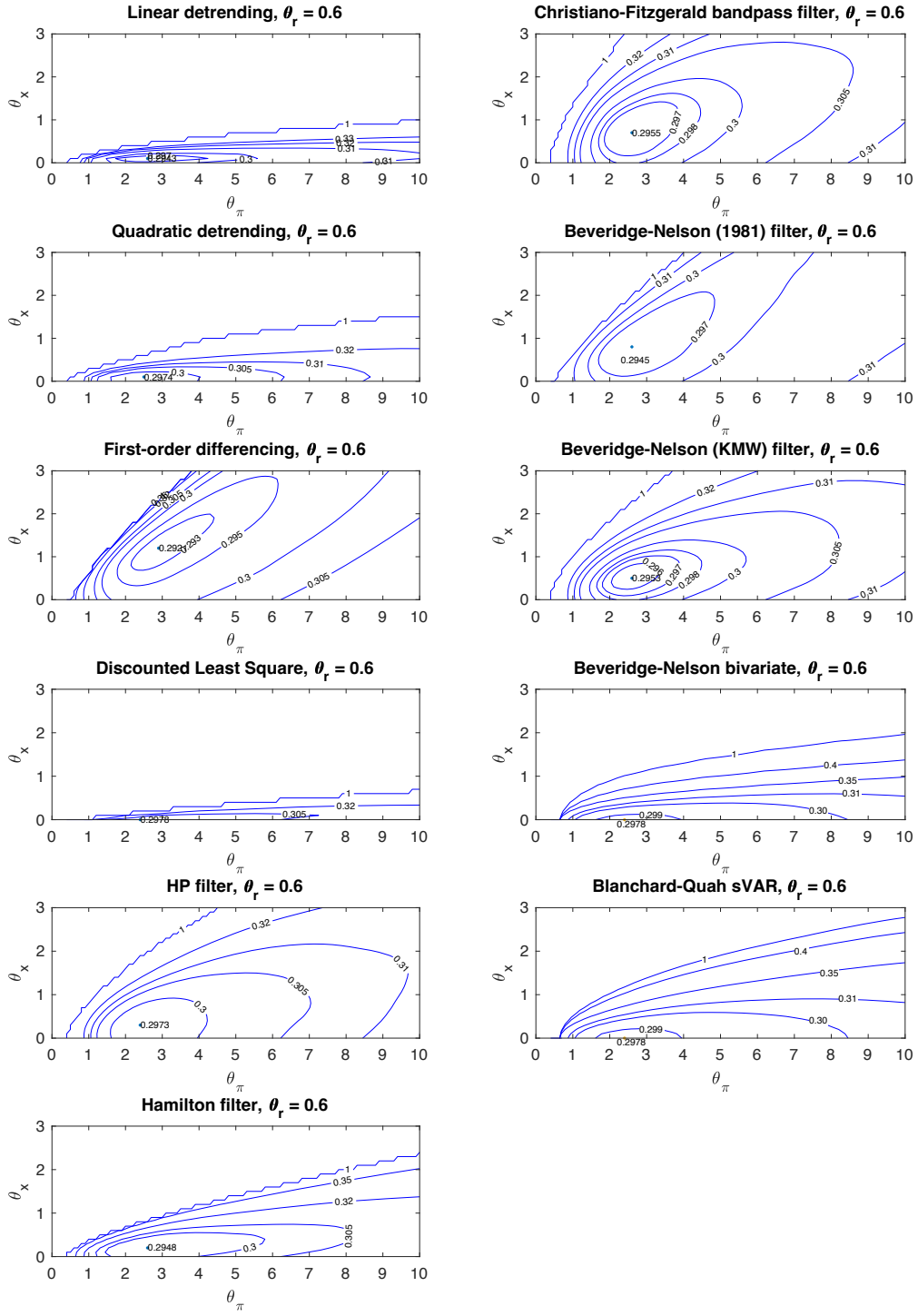
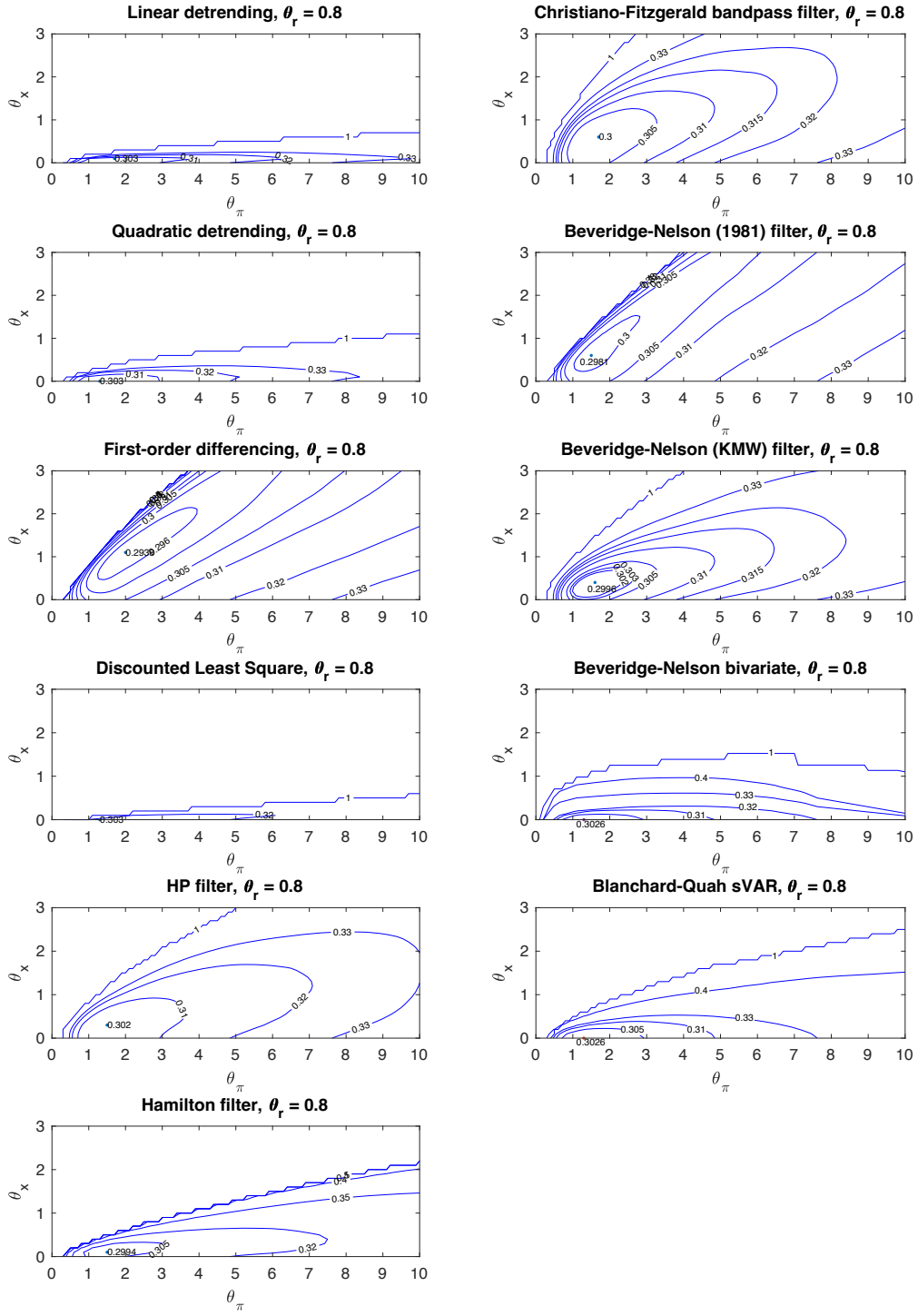


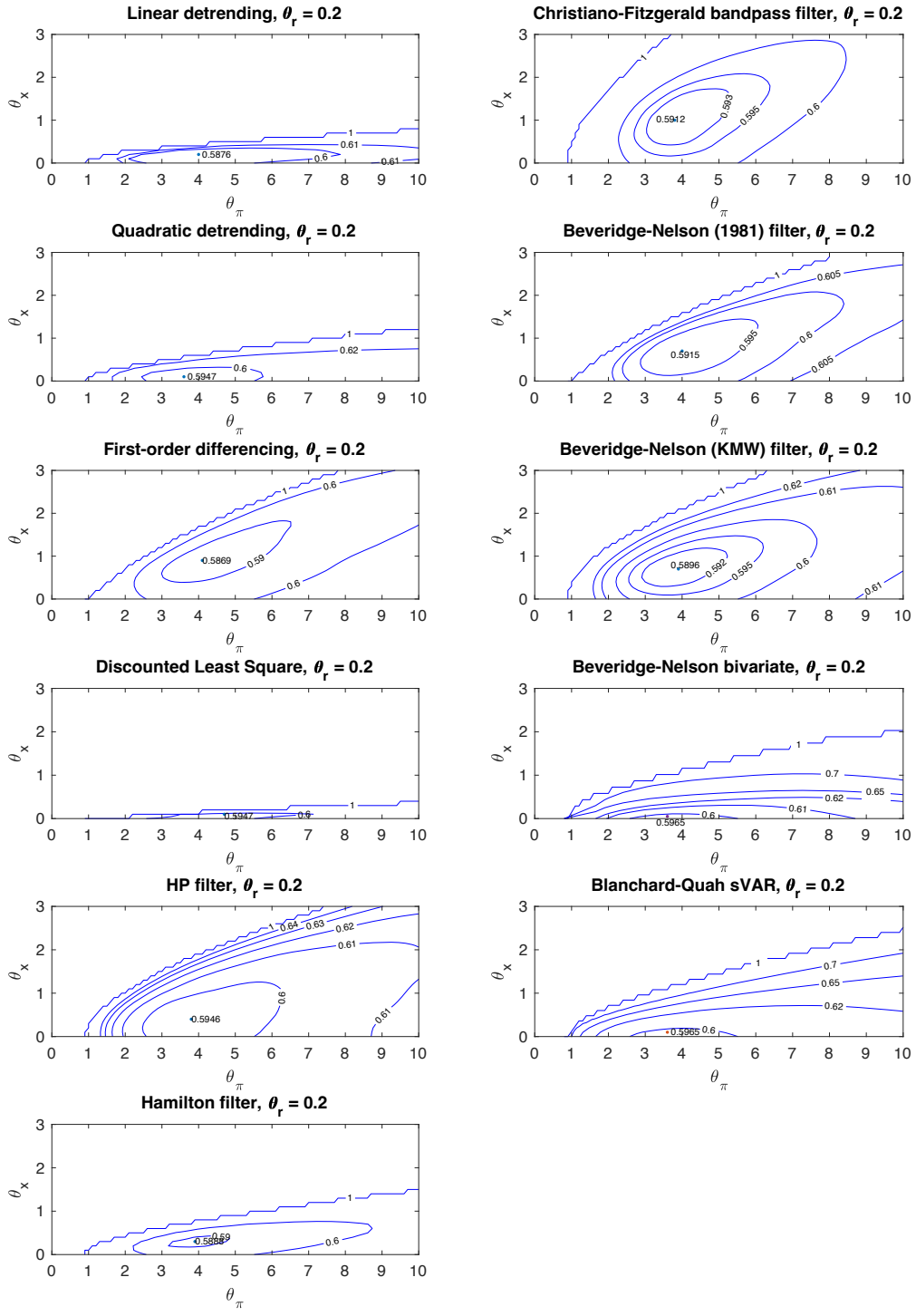
**Figure S.8: 4% INFLATION TARGET: ISO-LOSS CONTOURS, $\theta_r = 0.8$**



# 5 Dual mandate

This section complements Appendix K of the paper, which reports the iso-loss contours for the remaining degrees of inertia.

**Figure S.9: DUAL MANDATE: ISO-LOSS CONTOURS,** $\theta_r = 0.2$



## Figure S.10: DUAL MANDATE: ISO-LOSS CONTOURS, $\theta_r = 0.6$

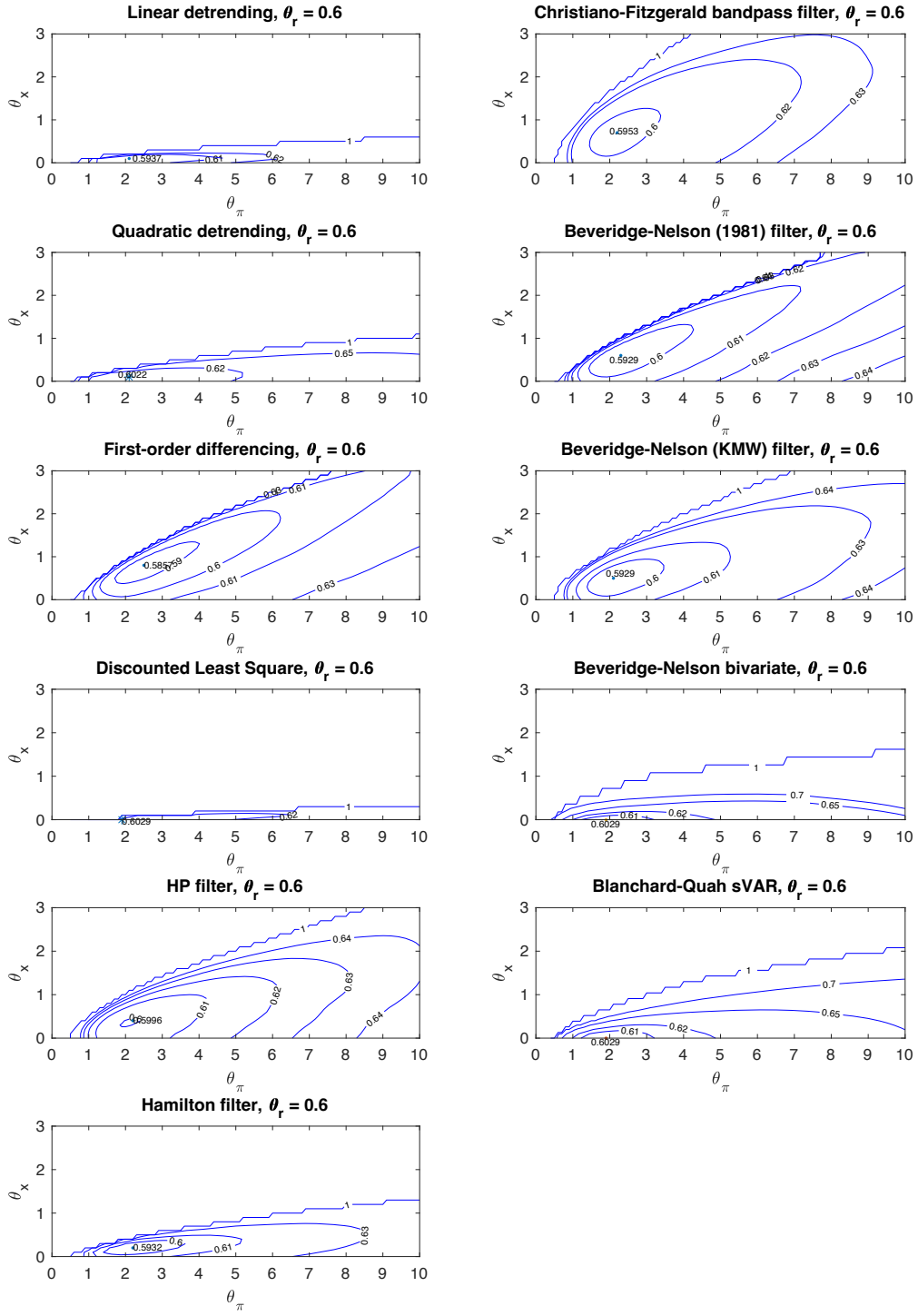
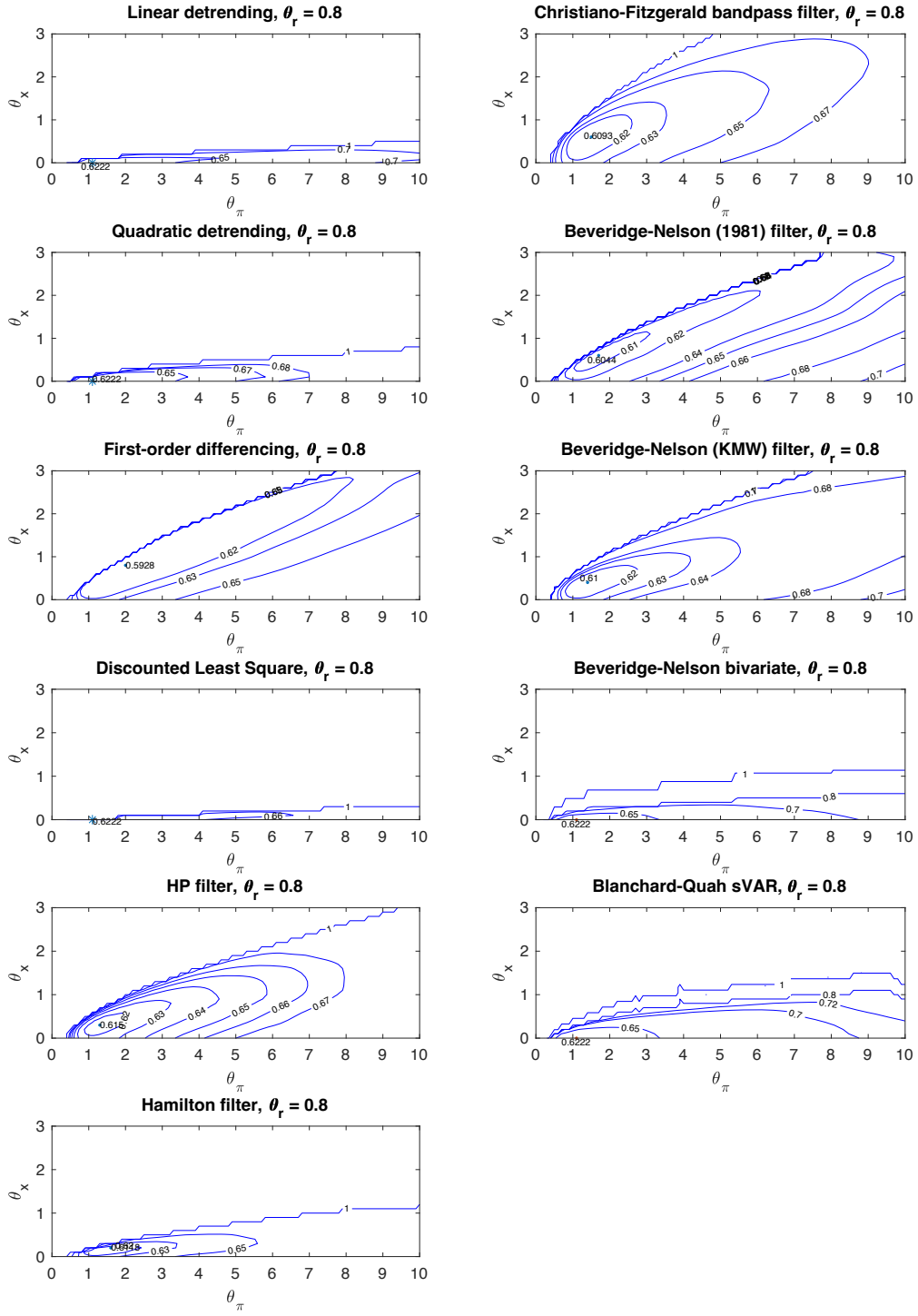


**Figure S.11: DUAL MANDATE: ISO-LOSS CONTOURS, $\theta_r = 0.8$**



# 6 A deterministic underlying trend

This section complements Appendix M.1 of the paper, which reports the iso-loss contours for the remaining degrees of inertia.

**Figure S.12: DETERMINISTIC TREND: ISO-LOSS CONTOURS, $\theta_r = 0.2$**

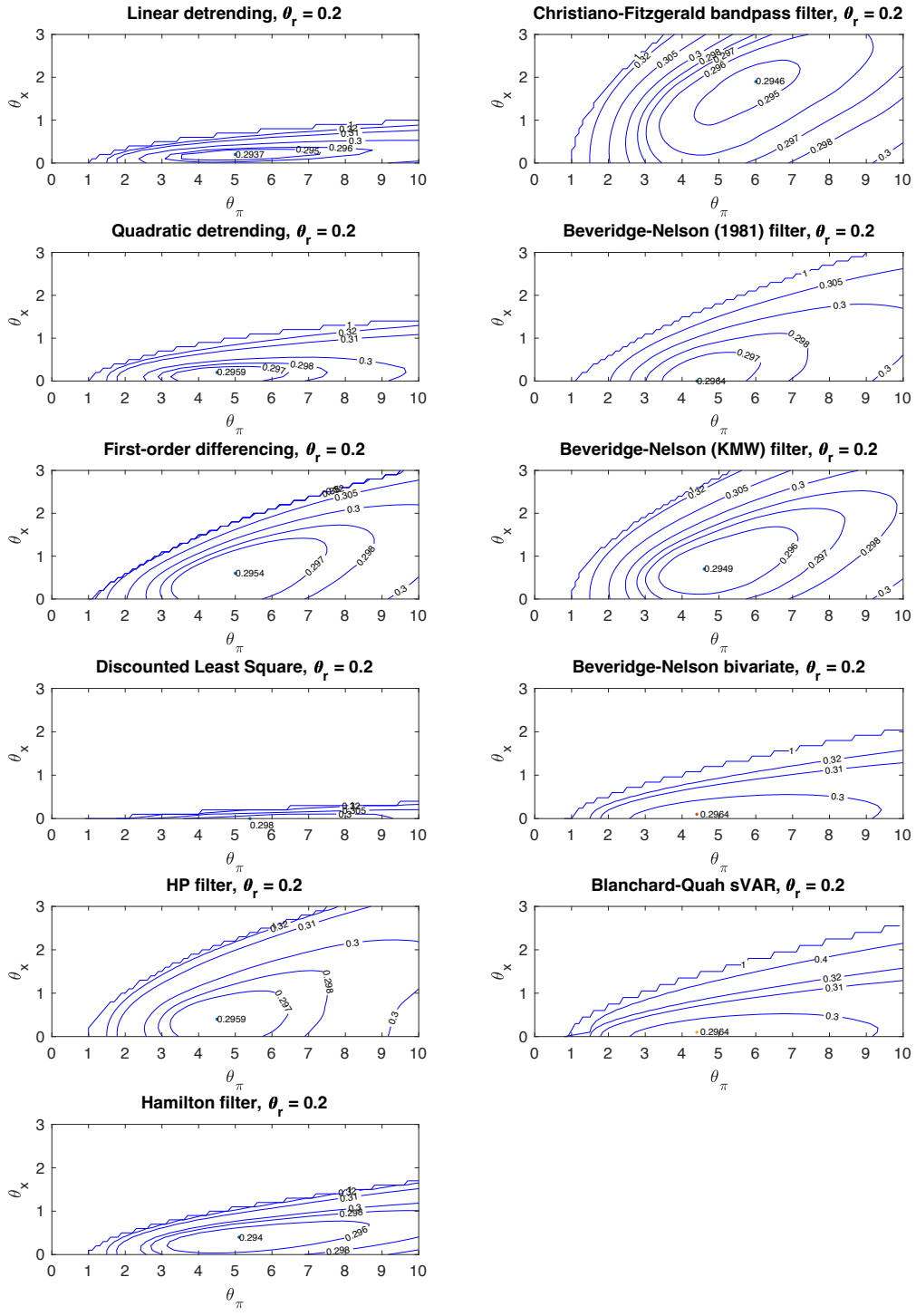


**Figure S.13: DETERMINISTIC TREND: ISO-LOSS CONTOURS, $\theta_r = 0.6$**

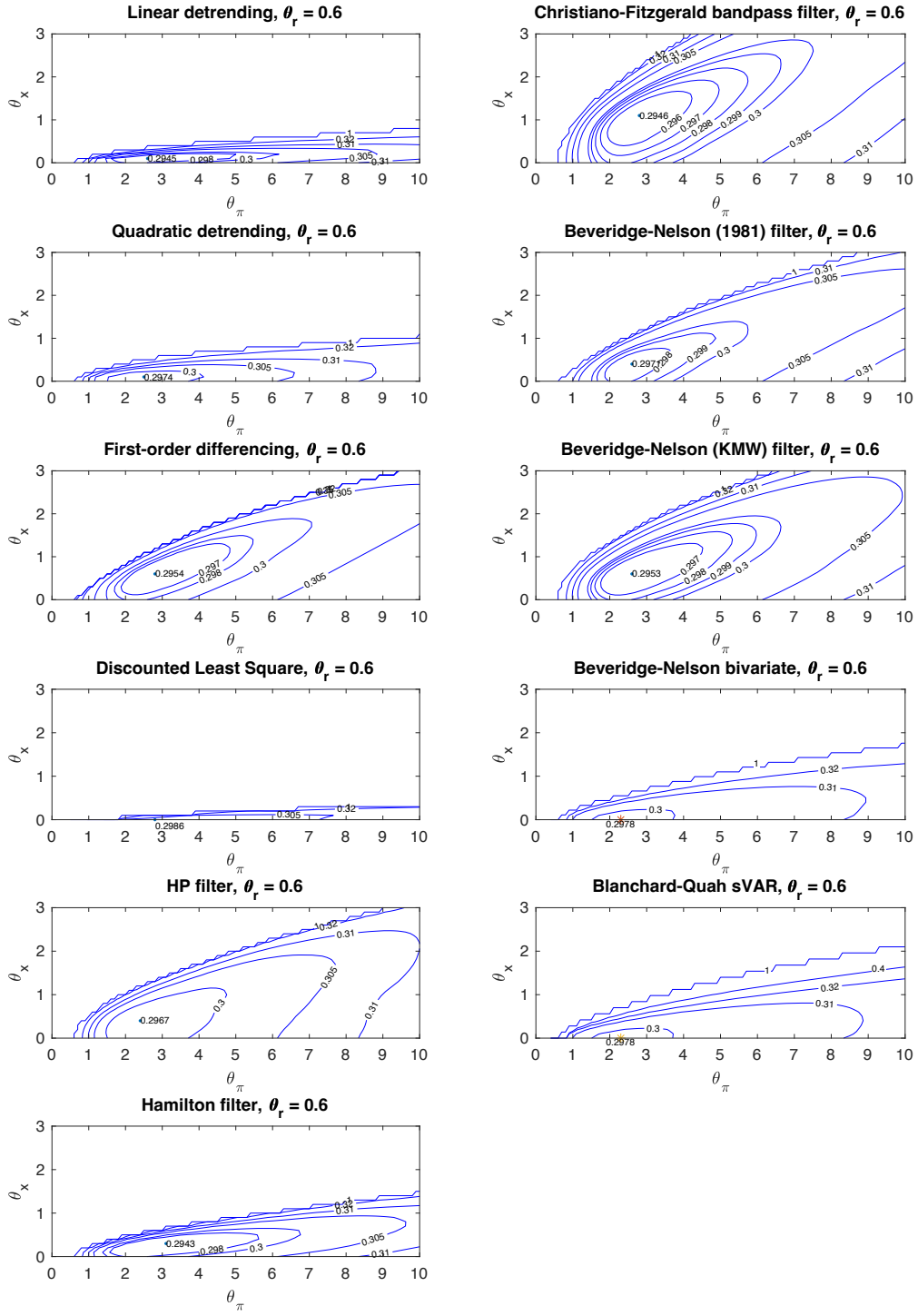
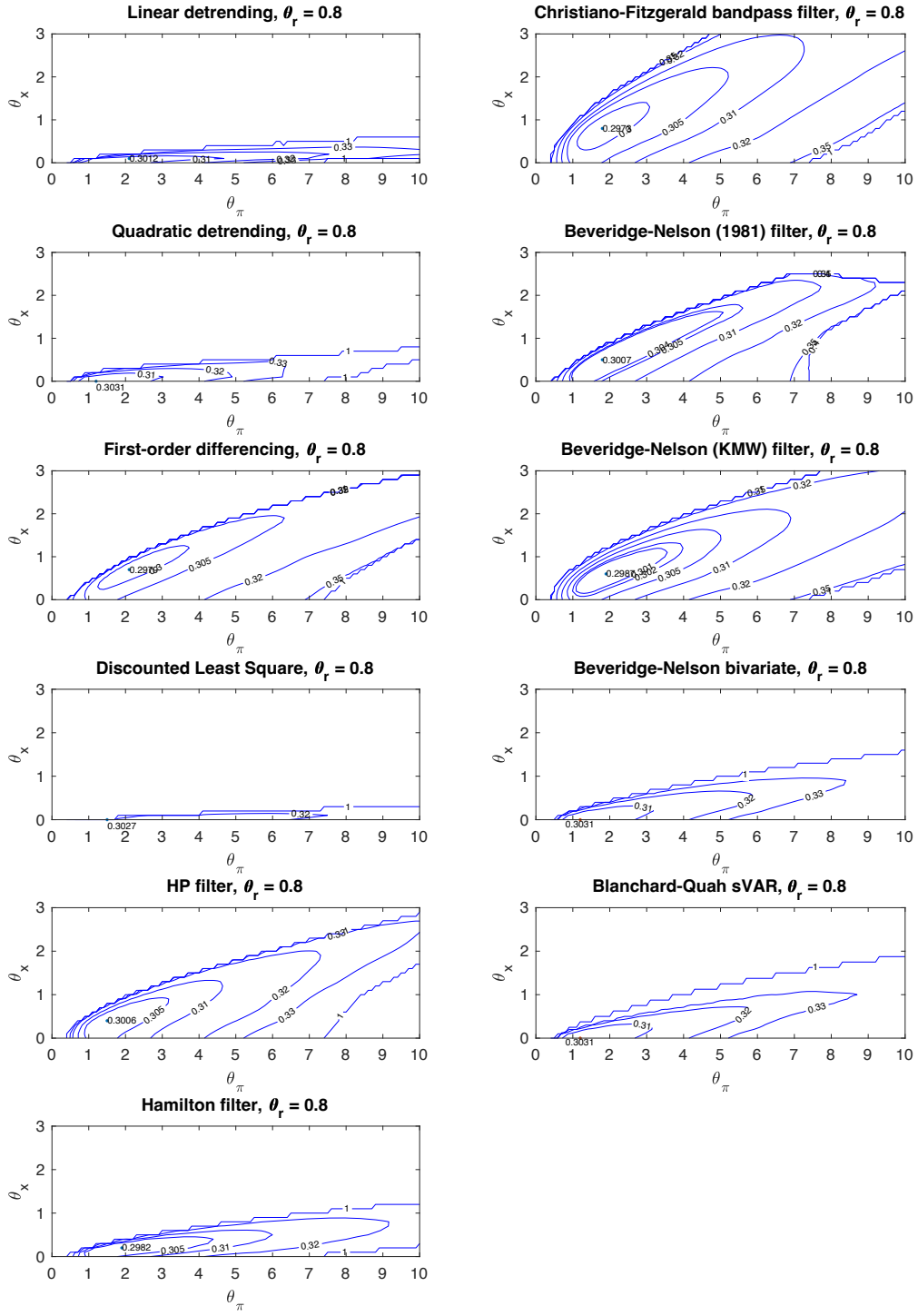


**Figure S.14: DETERMINISTIC TREND: ISO-LOSS CONTOURS, $\theta_r = 0.8$**



# 7 Both level and growth shocks to the trend

This section complements Appendix M.2 of the paper, which reports the iso-loss contours for the remaining degrees of inertia.

**Figure S.15: LEVEL AND GROWTH SHOCKS TO THE TREND: ISO-LOSS CONTOURS, $\theta_r = 0.2$**

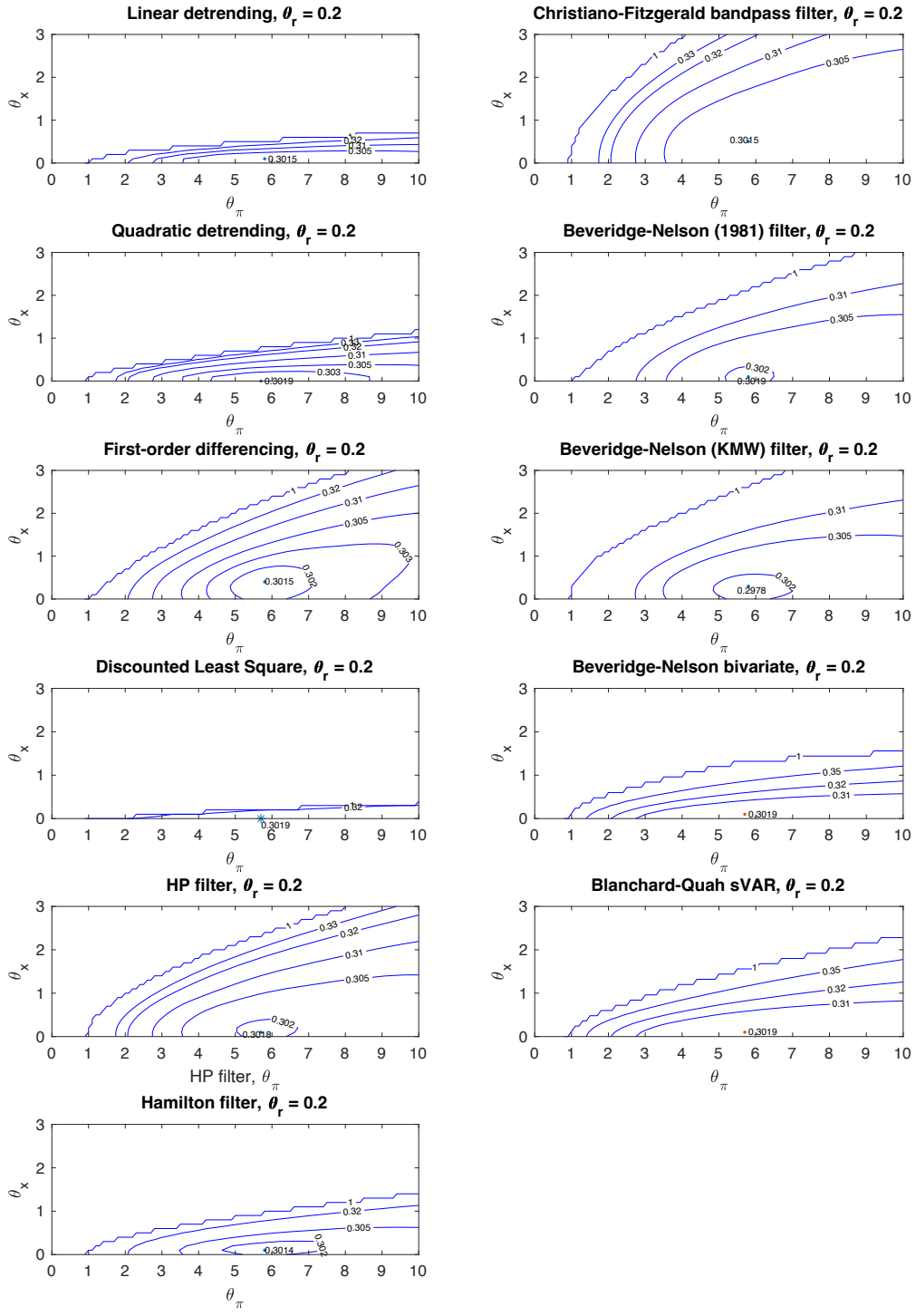


**Figure S.16: LEVEL AND GROWTH SHOCKS TO THE TREND: ISO-LOSS CONTOURS, $\theta_r = 0.6$**

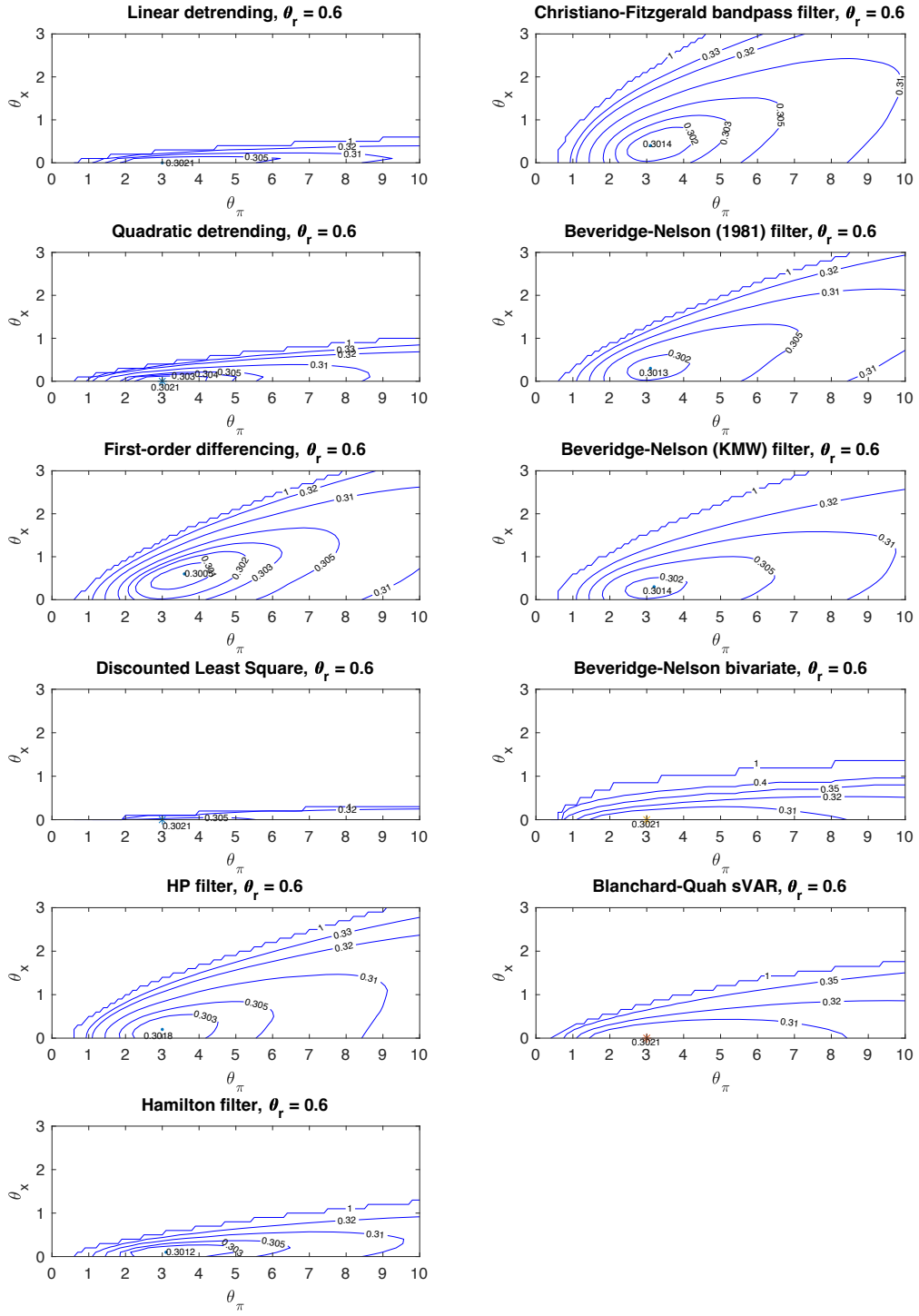


**Figure S.17: LEVEL AND GROWTH SHOCKS TO THE TREND: ISO-LOSS CONTOURS, $\theta_r = 0.8$**

